Supervised Clustering of Label Ranking Data

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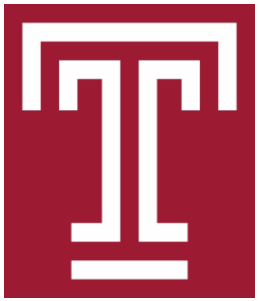
SIAM SDM 2012, Anaheim, California, USA



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    - Performance Measures
    - Related Work
  - Supervised clustering in context of Label Ranking
    - Motivation
    - Performance Measures
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  - Baseline Approaches
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- Empirical Evaluation
  - Experiments on Synthetic Data
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Introduction

- Label Ranking Setup: L = 5 labels

	Costumer Features					Product
	Features (**x**)					*y*
1	0	21.2	-1.2	1	12	1
2	1	-1.8	-1.1	0	14	3
3	1	-2.5	0.3	1	8	3
4	0	7.5	-0.2	1	11	2

- Costumer Features: age, gender, how often they buy from us, how much on average they spend, etc.

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Introduction

- Label Ranking Setup: L = 5 labels

	Costumer Features (x)					Product Ranking (π)				
1	0	21.2	-1.2	1	12	1	4	2	5	3
2	1	-1.8	-1.1	0	14	3	2	5	1	4
3	1	-2.5	0.3	1	8	3	2	1	4	5
4	0	7.5	-0.2	1	11	2	1	5	4	3

$\pi = 1\ 4\ 2\ 5\ 3$, pairwise label preferences: $1 > 4 > 2 > 5 > 3$

- Goal: Learn a model that maps instances $\mathbf{x}$ to a total label order π

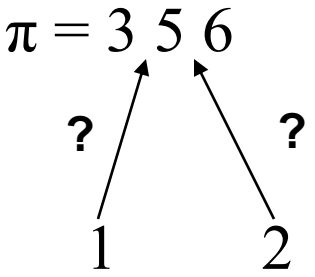
$$D = \{(\mathbf{x}_n, \pi_n), n = 1 \ldots N\} \qquad h : \mathbf{x}_n \rightarrow \pi_n$$

Introduction

- Label Ranking: Missing Information

	Features (x)					Label Ranking (π)				
1	0	21.2	-1.2	1	12	1	4			
2	1	-1.8	-1.1	0	14	3	5	4		
3	1	-2.5	0.3	1	8	2	1	4		
4	0	7.5	-0.2	1	11	1	5	3		

Partial Ranking



Introduction

- Label Ranking: Performance Measure

Notation: $\pi(i)$ the class label at i-th position in the order
$\pi^{-1}(j)$ the position of the y_j class label in the order

Distance between two rankings: true ranking (π) and predicted ranking (ρ):

$$d_\tau(\pi,\rho) = \left|\{(y_i, y_j) : \pi_n^{-1}(y_i) > \pi_n^{-1}(y_j)\} \mid \rho_n^{-1}(y_j) > \rho_n^{-1}(y_i)\}\right|$$

Kendall tau distance - counts the number of discordant label pairs

Given Data set: $D = \{(\mathbf{x}_n, \pi_n), n = 1 \ldots N\}$

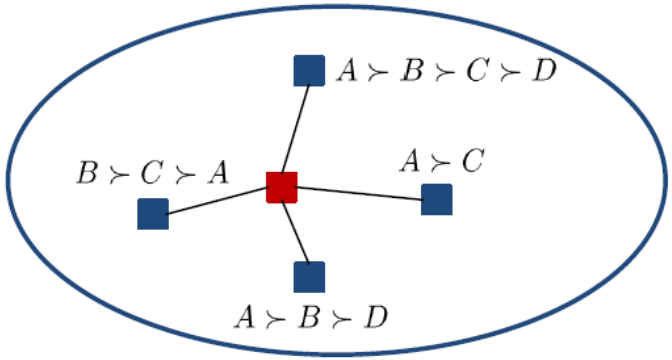
Label Ranking Loss: $loss_{LR} = \frac{1}{N}\sum_{n=1}^{N} \frac{2 \cdot d_\tau(\pi_n, \hat{\pi}_n)}{L \cdot (L-1)}$

Introduction

- Label Ranking: Related Work

1. Map into classification - $L(L\text{-}1)/2$ classifiers
   - 1 $(d \text{ x } L)$ dimensional problem

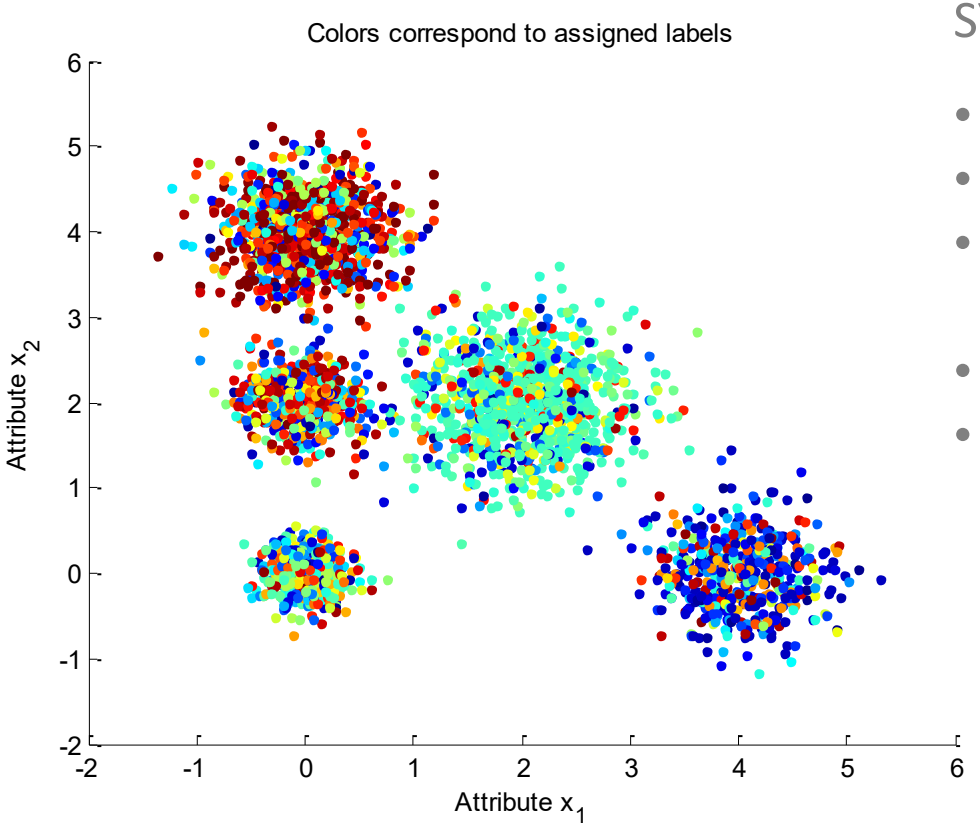
2. kNN based algorithms



3. Utility functions - Learn mappings $f_k : \mathbf{x} \rightarrow R, k = 1, \ldots, L$
   - Prediction: rank the utility scores

Introduction

- Label Ranking: Supervised Clustering



SYNTHETIC DATA

- 2 features
- 5 labels
- Each permutation represented with a color (similar color – similar rank)
- 5 natural clusters in feature space
- 3 natural clusters in label space

Introduction

- Label Ranking: Supervised Clustering

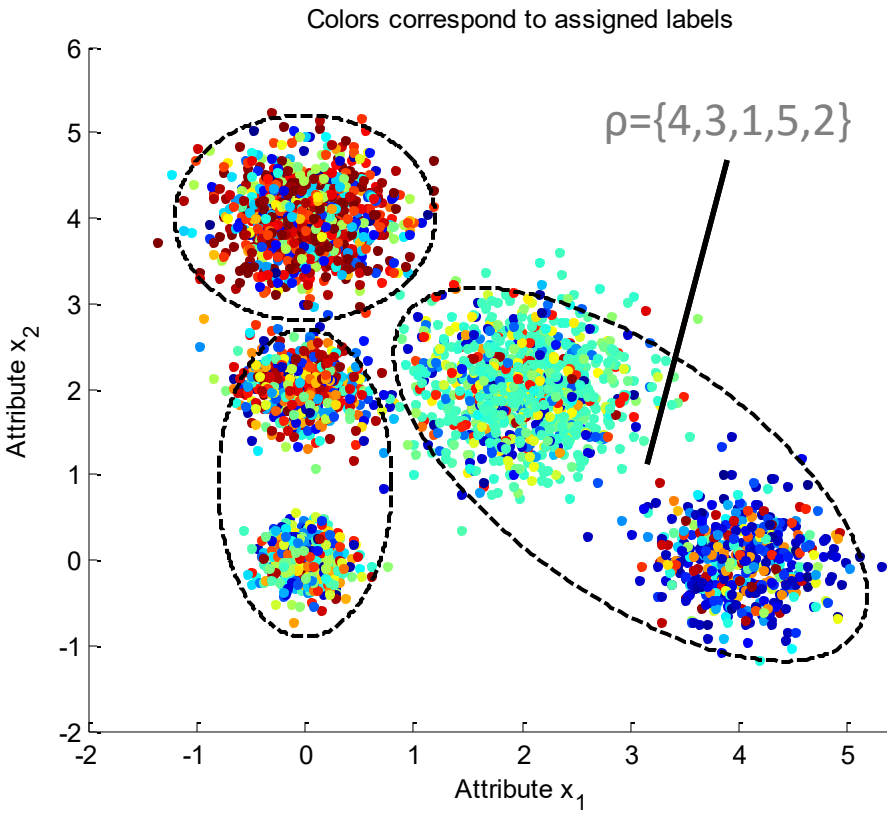
GOAL:

- Cluster data instances (customers) in the feature space by taking into consideration the assigned, potentially incomplete label rankings (product preferences)
- Such that the rankings of instances within a cluster are more similar to each other than to the rankings of instances in the other clusters
- Extract cluster centroid-rankings (preferences that represent each cluster uniquely)

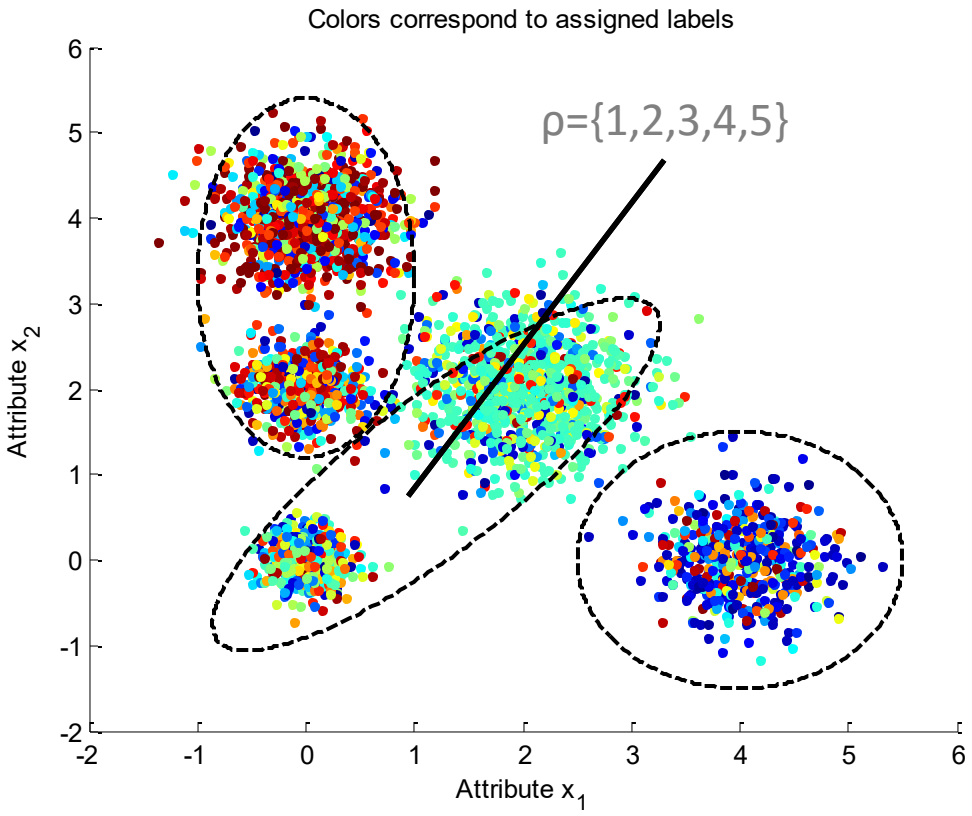
Introduction

- Label Ranking: Supervised Clustering

Traditional Clustering



Supervised Clustering



Introduction

- Label Ranking: Supervised Clustering

Example: **Target marketing**

- A company with several products would like to cluster its costumers (in feature space)
- Purpose: designing cluster-specific promotional material
- For each cluster, the company can make a different catalog, by promoting products in different order that best reflects the taste of its target costumers

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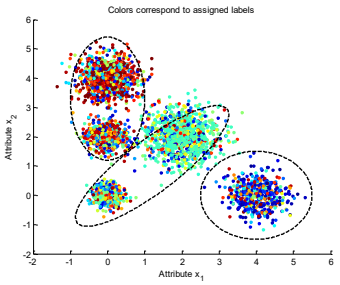
Introduction

- Label Ranking: Supervised Clustering

Performance Measures

- "Tightness" of clusters in label ranking space
  - How similar are the rankings of instances within the clusters
  - How far are cluster central ranking from cluster member rankings

$$\Delta_p = \frac{1}{K}\sum_{k=1}^{K}\frac{\sum_{n\in S_k} d_\tau(\pi_n, \rho_k)}{\mid S_k \mid},$$



- Happiness of new costumer when he receives the catalog by mail
  - How close is the cluster central ranking to true costumer ranking

$$loss_{LR} = \frac{1}{N}\sum_{n=1}^{N}\frac{2\cdot d_\tau(\pi_n, \hat{\pi}_n)}{L\cdot(L-1)}$$

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Approaches

- Heuristic Baselines

1. Cluster in Feature Space → Find Central Cluster Rankings

Kmeans → Mallows

2. Cluster in Label Ranking Space → Multi-Class Classification

Naïve → SVM
EBMS* → SVM

3. Add Label Rankings to Features → Unsupervised Clustering

Naïve Kmeans

4. 1-Rank (represent all data using one ranking)

* M. Meila and L. Bao, An exponential model for infinite rankings, Journal of Machine Learning Research, 11 (2010)

Approaches

- Plackett-Luce Mixture Model

The Plackett-Luce (PL) model is specified by a parameter vector $\boldsymbol{v} = (v_1, v_2, \ldots v_m) \in \mathbb{R}_+^m$:

$$\mathbf{P}(\pi \mid \boldsymbol{v}) = \prod_{i=1}^{m} \frac{v_{\pi(i)}}{v_{\pi(i)} + v_{\pi(i+1)} + \ldots + v_{\pi(m)}}$$

Reduces problem to learning a mapping $\boldsymbol{x} \mapsto \boldsymbol{v}$.

Example: $\boldsymbol{v} = (1, 4, 2)$, $\mathbf{P}(\pi \mid \boldsymbol{v}) = \frac{v_{\pi(1)}}{v_{\pi(1)}+v_{\pi(2)}+v_{\pi(3)}} \cdot \frac{v_{\pi(2)}}{v_{\pi(2)}+v_{\pi(3)}} \cdot 1$

1	2	3	0.0952
1	3	2	0.0476
2	1	3	0.1905
2	3	1	0.0571
3	1	2	0.3810
3	2	1	0.2286

Approaches

- Plackett-Luce Mixture Model (K clusters)

$$\mathbb{P}(\pi \mid \mathbf{x}) = \sum_{k=1}^{K} \mathbb{P}(k \mid \mathbf{x}) \cdot \mathbb{P}(\pi \mid k)$$

K clusters: $\lambda = \{\{\mathbf{m}_p\}_k, \mathbf{v}_k, k = 1, ..., K, \sigma_p\}$

Likelihood:

$$\mathbb{P}(\boldsymbol{\pi} \mid \lambda) = \prod_{n=1}^{N} \sum_{k=1}^{K} \mathbb{P}(k \mid \mathbf{x}) \cdot \prod_{i=1}^{L_n} \frac{\mathbf{v}_k(\pi_n(i))}{\sum_{j=i}^{L_n} \mathbf{v}_k(\pi_n(j))}$$

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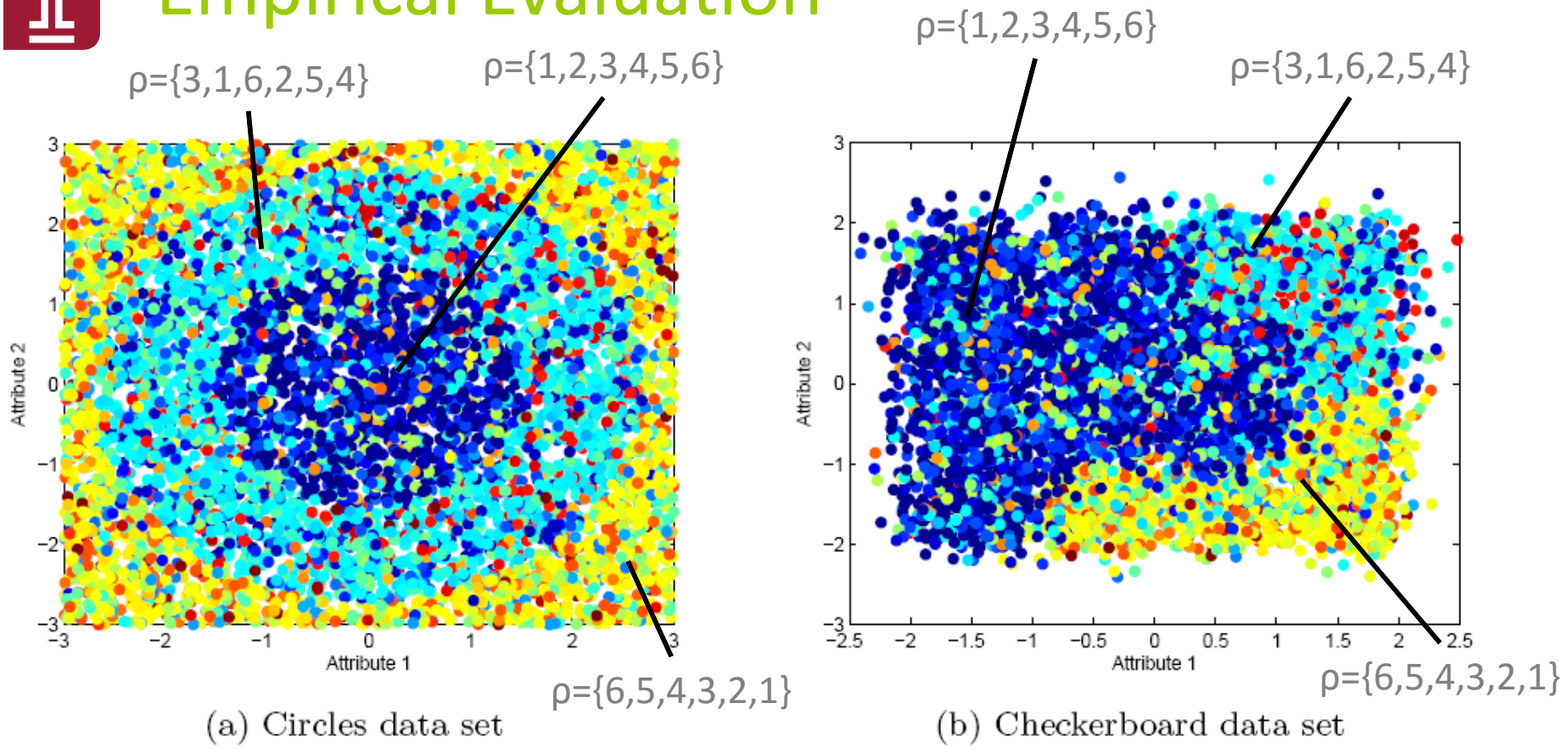
Empirical Evaluation



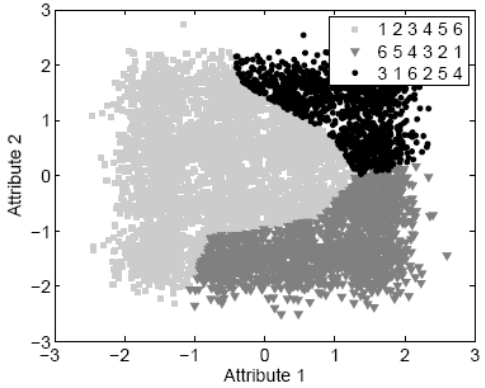
(a) Circles data set (b) Checkerboard data set

The resulting cluster central rankings

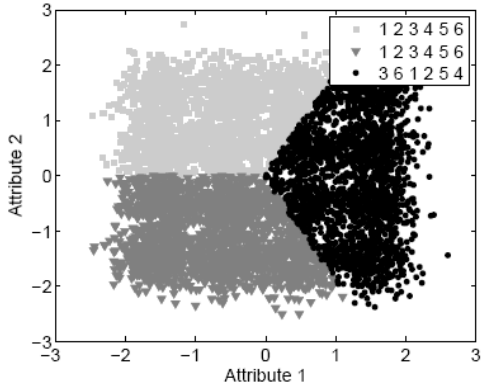
Data Set	complete ranking					30% missing labels					60% missing labels				
	Kmean *Mal*	Naive *Kmean*	Naive *SVM*	EBMS *SVM*	MM *PL*	Kmean *Mal*	Naive *Kmean*	Naive *SVM*	EBMS *SVM*	MM *PL*	Kmean *Mal*	Naive *Kmean*	Naive *SVM*	EBMS *SVM*	MM *PL*
checker	**123456**	**123456**	**123456**	132654	**123456**	**123456**	**123456**	**123456**	132546	**123456**	**123456**	**123456**	**123456**	132645	**123456**
$K = 3$	123456	**654321**	**654321**	653421	**654321**	123456	**654321**	**654321**	132645	**654321**	316254	**654321**	124356	132645	**654321**
	361524	423516	213456	653421	**316254**	361254	136254	124356	132645	**316254**	316452	123645	123546	132546	**316254**
circle	361254	**123456**	136254	**123456**	**123456**	361524	132654	312654	653421	**123456**	361524	635124	316524	316254	**123456**
$K = 3$	316254	**654321**	**654321**	132546	**654321**	361524	**654321**	**654321**	653421	**654321**	361254	316254	361254	316254	**654321**
	361524	**316254**	**316254**	312654	**316254**	361524	654321	**316254**	132654	**316254**	361524	**316254**	**316254**	**316254**	**316254**

Empirical Evaluation

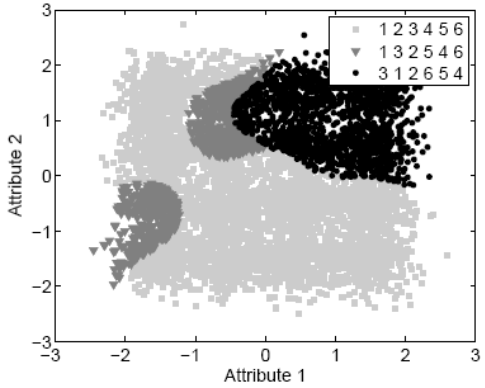
$$\Delta_p = \frac{1}{K}\sum_{k=1}^{K}\frac{\sum_{n\in S_k} d_\tau(\pi_n, \rho_k)}{|\,S_k\,|}$$



(a) $\underset{PL}{\text{MM}}$, $\Delta_p = .526$

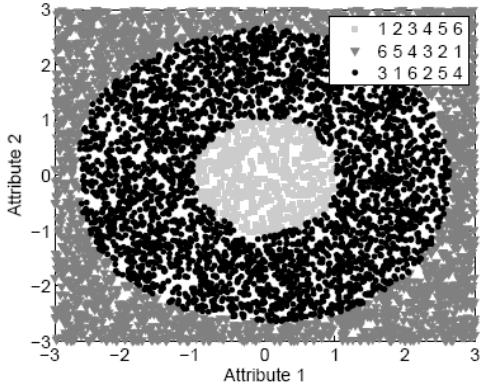


(b) $\underset{Mal}{\text{Km}}$, $\Delta_p = .771$

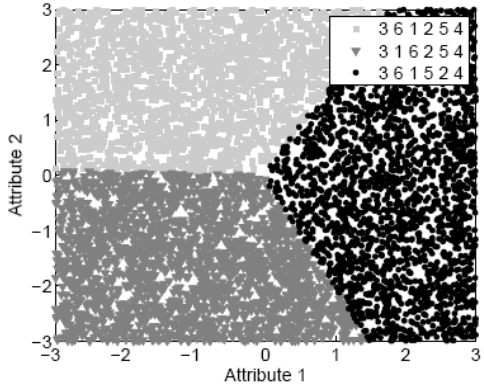


(c) $\underset{SVM}{\text{ebms}}$, $\Delta_p = .617$

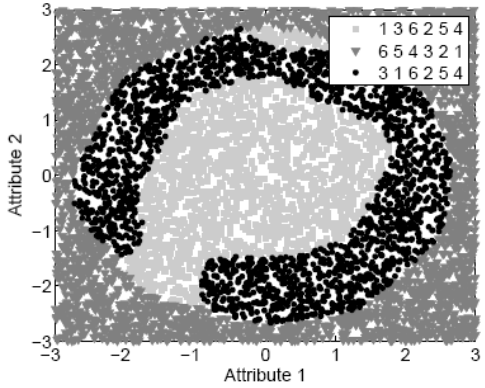
Clustering performance comparison on *checker* dataset ($K = 3$)



(a) $\underset{PL}{\text{MM}}$, $\Delta_p = .532$



(b) $\underset{Mal}{\text{Km}}$, $\Delta_p = .831$

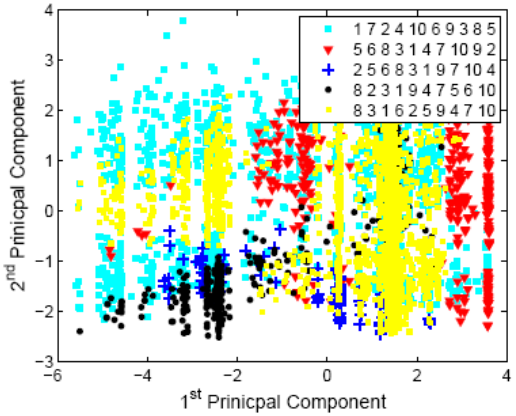


(c) $\underset{SVM}{\text{naive}}$, $\Delta_=.552$

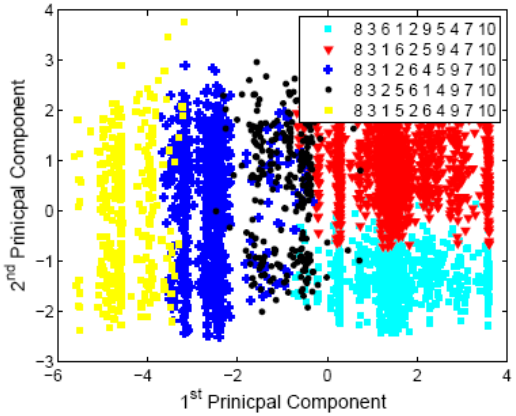
Clustering performance comparison on *circle* dataset ($K = 3$)

Empirical Evaluation

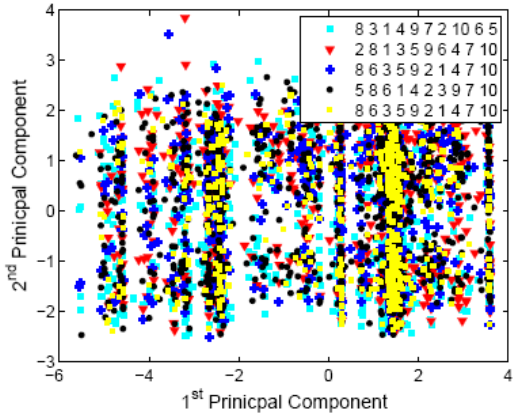
- Sushi Data Set (L=10)



(a) $\underset{PL}{\text{MM}}$, $\Delta_p = .641$



(b) $\underset{Mal}{\text{Km}}$, $\Delta_p = .695$



(c) $\underset{SVM}{\text{ebms}}$, $\Delta_p = .645$

Clustering performance comparison on *sushi* dataset ($K = 5$)

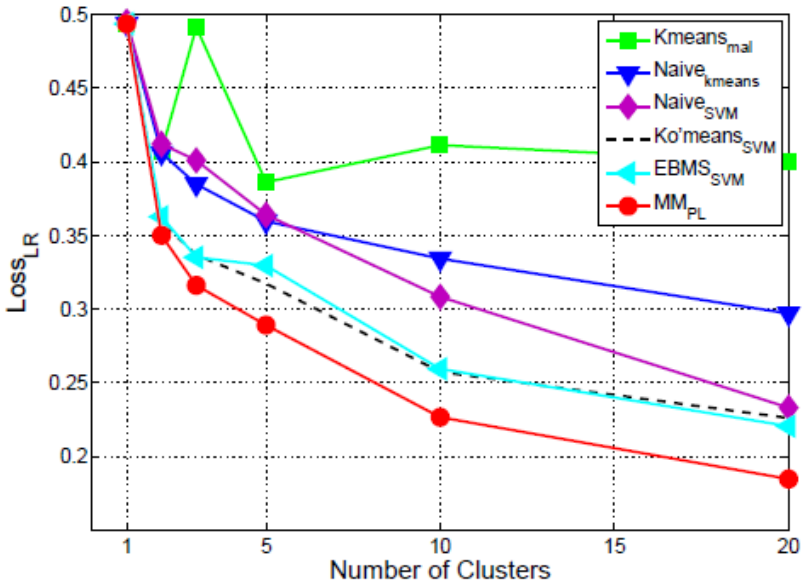
The resulting cluster central rankings for Sushi data set $K = 5$, no missing labels

1 Rank		Km $_{Mal}$	naive $_{Km}$	naive $_{SVM}$	ebms $_{SVM}$	MM $_{PL}$
		8 3 6 1 2 9 5 4 7 10	5 2 1 8 6 4 3 9 7 10	2 5 6 8 3 1 9 7 10 4	8 3 1 4 9 7 2 10 6 5	1 7 2 4 10 6 9 3 8 5
		8 3 1 6 2 5 9 4 7 10	1 7 4 2 3 10 9 8 6 5	5 6 8 3 1 4 9 2 7 10	2 8 1 3 5 9 6 4 7 10	5 6 8 3 1 4 7 10 9 2
8 3 1 6 2 5 9 4 7 10		8 3 1 2 6 4 5 9 7 10	8 3 9 1 2 6 4 7 10 5	8 3 6 5 9 2 1 4 10 7	8 6 3 5 9 2 1 4 7 10	2 5 6 8 3 1 9 7 10 4
		8 3 2 5 6 1 4 9 7 10	8 5 2 6 3 9 1 7 4 10	8 5 6 2 1 3 9 4 10 7	5 8 6 1 4 2 3 9 7 10	8 2 3 1 9 4 7 5 6 10
		8 3 1 5 2 6 4 9 7 10	8 5 3 6 1 4 9 2 7 10	1 8 3 9 2 4 7 10 6 5	8 6 3 5 9 2 1 4 7 10	8 3 1 6 2 5 9 4 7 10
Δ_p	.695	.778	.652	.645	**.641**	

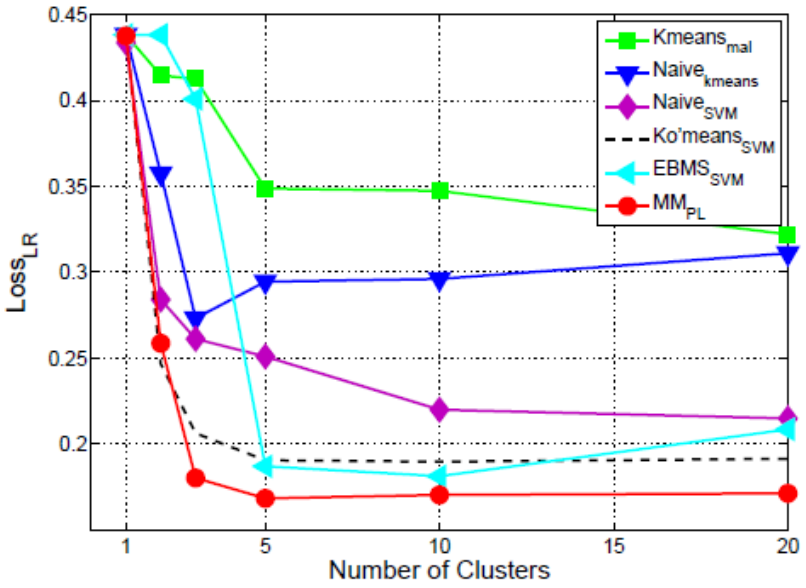
Empirical Evaluation

name	domain	N	d	L	1-Rank $loss_{LR}$	name	domain	N	d	L	1-Rank $loss_{LR}$
circles	S	7,000	2	6	.393	checker	S	7,200	2	6	.398
authorship	SS (A)	841	70	4	.269	iris	SS (B)	150	4	3	.451
bodyfat	SS (B)	252	7	7	.450	pendigits	SS (A)	10,992	16	10	.383
calhousing	SS (B)	20,640	4	4	.380	segment	SS (A)	2,310	18	7	.419
cpu-small	SS (B)	8,192	6	5	.431	stock	SS (B)	950	5	5	.445
elevators	SS (B)	16,599	9	9	.435	vehicle	SS (A)	846	18	4	.335
fried	SS (B)	40,769	9	5	.493	vowel	SS (A)	528	10	11	.334
glass	SS (A)	214	9	6	.171	wine	SS (A)	178	13	3	.234
housing	SS (B)	506	6	6	.412	wisconsin	SS (B)	194	16	16	.485
cold	biology	2,465	24	4	.401	heat	biology	2,465	24	6	.464
diau	biology	2,465	24	7	.348	spo	biology	2,465	24	11	.441
dtt	biology	2,465	24	4	.438	sushi	food	5,000	11	10	.392

Empirical Evaluation



(a) Fried Data $Loss_{LR}$



(b) Elevator Data $Loss_{LR}$

Empirical Evaluation

Label Ranking Supervised Clustering performance comparison in terms of $loss_{LR}$

	complete ranking					30% missing labels					60% missing labels				
Data Set	Km *Mal*	naive *Km*	naive *SVM*	ebms *SVM*	MM *PL*	Km *Mal*	naive *Km*	naive *SVM*	ebms *SVM*	MM *PL*	Km *Mal*	naive *Km*	naive *SVM*	ebms *SVM*	MM *PL*
checker	.340	.287	.287	.373	**.250**	.340	.300	.287	.377	**.249**	.342	.308	.413	.393	**.260**
circle	.392	.444	.260	.281	**.253**	.392	.410	.265	.280	**.258**	.392	.396	.374	.395	**.261**
fried	.413	.334	.272	.263	**.226**	.412	.342	.291	.340	**.236**	.422	.395	.317	.375	**.251**
calhousing	.364	.424	.450	.359	**.349**	.365	.463	.467	.379	**.352**	.365	.501	.469	.390	**.349**
elevators	.347	.300	.220	.182	**.171**	.334	.301	.272	.248	**.210**	.336	.308	.293	.271	**.214**
pendigits	.290	.308	**.148**	.314	.172	.289	.287	.197	.326	**.181**	.291	.337	.236	.349	**.202**
cpu-small	.362	.374	.407	.403	**.356**	.363	.389	.390	.399	**.356**	.365	.413	.407	.387	**.360**
segment	.214	.188	**.080**	.185	.146	.214	.197	**.115**	.184	.145	.213	.229	.202	.226	**.156**
wisconsin	.451	.443	.486	.453	.433	.455	.443	.487	.462	.435	.465	.461	.490	.469	**.444**
vowel	.253	.274	.237	.221	.209	.255	.276	.266	.239	**.211**	.265	.314	.305	.256	**.242**
vehicle	.132	.113	.095	.124	.090	.131	.165	.080	.155	**.095**	.136	.210	.142	.174	**.109**
stock	.213	.218	**.138**	.235	.140	.217	.227	.175	.241	**.148**	.222	.259	.273	.310	**.162**
iris	.091	.059	.080	.171	.061	.093	.142	.100	.174	**.088**	.136	.188	.196	.256	**.127**
glass	.110	.113	.092	.112	.108	.117	.173	**.098**	.125	.114	.139	.211	.189	.144	**.130**
authorship	.072	.066	.101	.175	.079	.073	**.072**	.109	.179	.075	.082	**.077**	.193	.221	.082
bodyfat	.451	.454	.461	**.419**	.430	.455	.459	.464	.453	.436	.469	.464	.491	.458	**.444**
wine	.050	.058	.051	.106	**.047**	.051	.163	.059	.122	**.045**	.087	.220	.104	.133	**.076**
housing	.376	.407	**.331**	.366	.339	.377	.421	.381	.412	**.357**	.397	.436	.384	.426	**.365**
cold	.395	.408	.416	.426	**.386**	.405	.404	.448	.437	**.393**	.404	.418	.434	.438	**.398**
heat	.438	.453	.477	.465	**.433**	.440	.451	.495	.472	**.437**	.447	.456	.477	.485	**.439**
diau	.336	.385	.379	.356	**.327**	.339	.376	.408	.366	**.338**	.345	.378	.362	.382	**.343**
dtt	.416	.432	.471	.447	**.410**	.422	.428	.464	.449	**.417**	.424	.432	.466	.453	**.422**
spo	**.431**	.440	.484	.463	**.431**	**.434**	.445	.487	.474	.435	.443	.471	.481	.478	**.440**
sushi	.372	.383	.379	.366	**.332**	.369	.374	.414	.372	**.335**	.379	.394	.382	.477	**.343**
average	.305	.307	.284	.303	**.257**	.306	.321	.301	.319	**.264**	.315	.345	.336	.348	**.276**
time (s)	10.1	9.3	1300	13K	234	13.5	10.9	3190	13.2K	228	13.8	11.6	3730	13.3K	188

Empirical Evaluation

Label Ranking Prediction performance comparison in terms of label rank loss $loss_{LR}$

	complete ranking							30% missing labels					60% missing labels				
Data Set	1NN	IB $_{CPS}$	IB $_{PL}$	IB $_{Mal}$	Lin $_{LL}$	Lin $_{PL}$	MM $_{PL}$	IB $_{PL}$	IB $_{Mal}$	Lin $_{LL}$	Lin $_{PL}$	MM $_{PL}$	IB $_{PL}$	IB $_{Mal}$	Lin $_{LL}$	Lin $_{PL}$	MM $_{PL}$
fried	.248	.201	.202	**.196**	.344	.347	.294	**.216**	.219	.345	.348	.300	**.257**	.312	.349	.350	.313
calhousing	.386	.344	.337	**.336**	.449	.435	.351	**.345**	.348	.450	.436	.353	**.376**	.415	.453	.437	.356
elevators	.261	**.224**	.234	.226	.306	.305	.244	**.238**	.244	.309	.306	.249	**.263**	.322	.311	.307	.265
pendigits	**.142**	.140	.143	.139	.364	.351	.211	**.152**	.159	.366	.354	.215	**.169**	.230	.368	.355	.219
cpu-small	.392	.347	.344	**.336**	.378	.361	.348	.351	**.350**	.379	.369	**.350**	.371	.408	.380	.370	**.355**
segment	**.073**	.077	.089	.089	.241	.227	.132	**.111**	.117	.242	.228	.135	**.124**	.154	.242	.226	.149
wisconsin	.458	.492	.435	.419	.403	**.401**	.413	.434	.427	.409	**.403**	.418	.445	.443	.422	**.412**	.430
vowel	**.110**	.117	.158	.148	.313	.317	.169	.217	.178	.316	.318	**.173**	.234	.240	.329	.327	**.231**
vehicle	.097	.087	.082	**.079**	.126	.115	.083	**.093**	.095	.129	.118	**.093**	.113	.131	.128	.125	**.102**
stock	.107	.118	.106	.108	.283	.268	**.105**	.134	.133	.284	.269	**.130**	.154	.169	.285	.272	**.151**
iris	.043	.049	.040	**.037**	.189	.171	.047	**.059**	.066	.199	.172	.064	**.103**	.118	.208	.174	.105
glass	**.075**	.102	.098	.094	.107	.106	.090	.107	.111	.111	.108	**.103**	**.115**	.140	.124	.118	.120
authorship	.080	.063	.063	**.062**	.182	.175	**.062**	.068	.072	.189	.178	**.065**	.085	.108	.200	.190	**.081**
bodyfat	.472	.436	.442	.438	.422	**.404**	.423	.463	.448	.430	**.406**	.430	.466	.460	.431	**.429**	.439
wine	.068	.039	.040	.036	.067	.059	**.034**	.041	.042	.071	.064	**.038**	.061	.077	.101	.079	**.059**
housing	.323	**.321**	.329	.334	.398	.385	.330	.355	.357	.404	.386	**.348**	.384	.385	.408	.391	**.364**
cold	.425	.389	.386	.387	.409	.399	**.385**	.398	.398	.410	.404	**.387**	.424	.423	.411	.405	**.397**
heat	.446	.434	.438	.435	.439	.438	**.430**	.445	.433	.441	.442	**.432**	.460	.455	.445	.443	**.435**
diau	.382	.336	.336	.332	.350	.349	**.329**	.338	.340	.353	.350	**.332**	.339	.365	.360	.352	**.338**
dtt	.440	.420	.418	.417	.421	.410	**.410**	.428	.430	.422	.417	**.414**	.447	.442	.423	**.419**	.423
spo	.440	.435	.438	.433	.437	.443	**.428**	.439	.444	.439	.445	**.434**	.441	.455	.444	.453	**.436**
sushi	.415	.347	.348	.349	.459	.462	**.337**	.363	.352	.461	.465	**.339**	.381	.374	.463	.468	**.342**
average	.267	.253	.250	**.247**	.322	.315	.255	**.263**	.264	.325	.318	**.263**	.282	.301	.331	.323	**.277**
avg. rank	4.73	3.23	3.41	**2.55**	6	5.27	2.82	2.36	2.77	4.32	3.77	**1.77**	2.45	3.64	3.95	3.14	**1.82**

Conclusion

- Conclusion

  - This paper presents the first attempt at supervised clustering of complex label rank data

  - We established several baselines for supervised clustering of label ranking data and proposed a Plackett-Luce (PL) mixture model specifically tailored for this application

  - We empirically showed the strength of the PL model by experiments on real-world and synthetic data

  - In addition to the supervised clustering scenario, we compared the PL model to the previously proposed label ranking algorithms in terms of predictive accuracy

THANK YOU