Temporally-Continuous Probabilistic Prediction using Polynomial Trajectory Parameterization

Zhaoen Su

Chao Wang

Henggang Cui

Nemanja Djuric

Carlos Vallespi-Gonzalez

David Bradley

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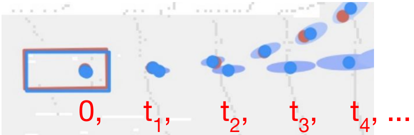
Representations for (probabilistic) trajectory prediction

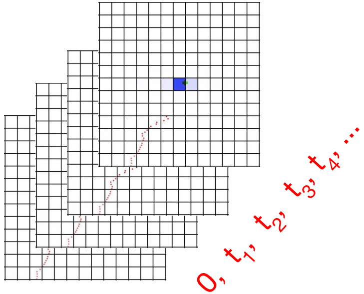
Discrete temporally ←- - - - - - - - - - - - - - - -→ *Continuous temporally*

Waypoints: spatial distributions for the SE3/SE2 transformation at each time-point

$$\mathscr{P} = \{(p(\mathbf{T}_t), p(\mathbf{R}_t))\}, t \in \{0, t_1, t_2, \ldots, T\}$$



Occupancy maps



Parametrize the time-varying distributions,
A univariate distribution for each component

$$P_v(t) = \mathscr{L}(v|\mu_v(t), b_v(t))$$

Each scalar element of the parameters governing the distribution is expressed by a polynomial

Means

$$\mu_v(t) = \sum_{n=0}^{N_{\mu_v}} a_{\mu_v,n} \left(\frac{t}{T}\right)^n$$

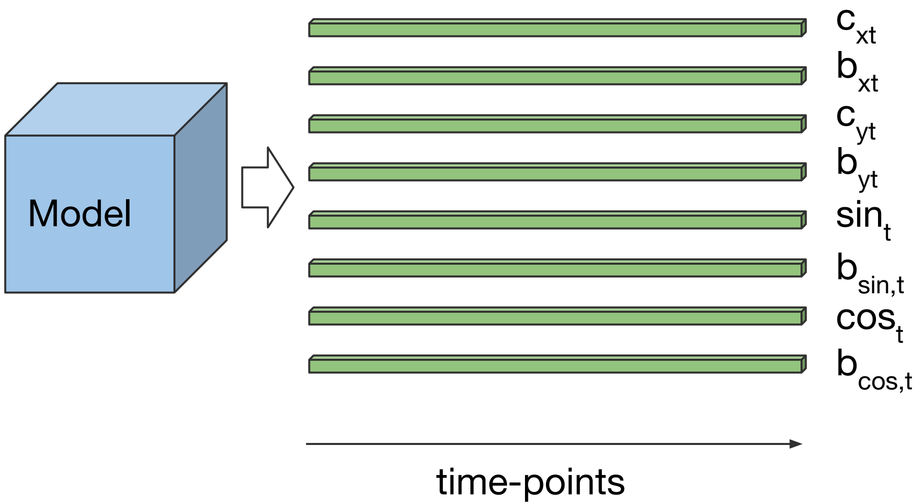
Diversity parameter

$$b_v(t) = \exp \sum_{n=0}^{N_{b_v}} a_{b_v,n} \left(\frac{t}{T}\right)^n$$

The representations in prediction learning

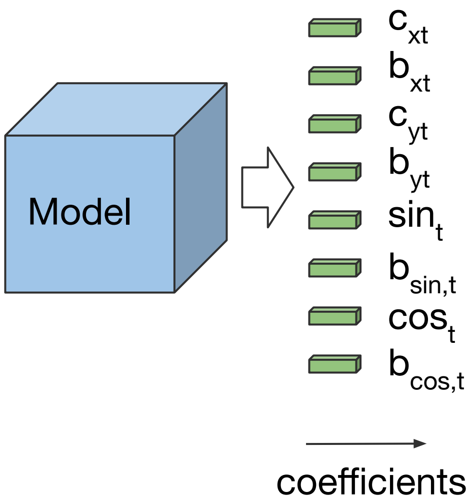
$$\boxed{v \in (c_{xt}, c_{y_t}, \sin\theta_t, \cos\theta_t)}$$

<u>Waypoints:</u> regress the movements for every time-point



Regress 628 values for every actor
for 8-sec prediction with 0.1s interval

<u>Polynomial:</u> regress the coefficients



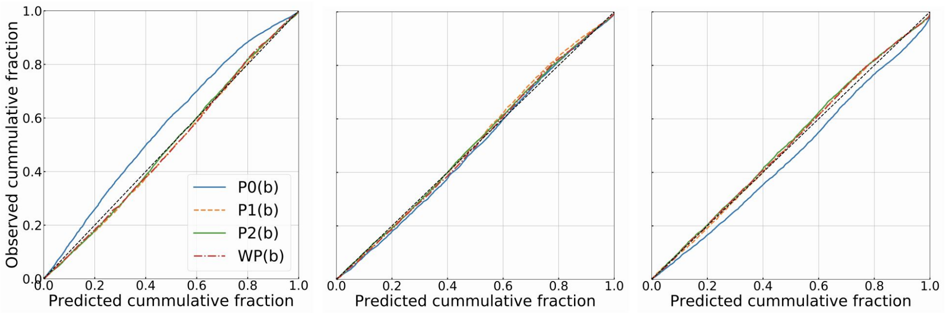
Regress 20 values (N_u=2, N_b=1)
for 8-sec prediction with infinitely small interval

Comparable to other representations

	Vehicles				Bicyclists				Pedestrians	
Method	**4s DE**	**8s DE**	**4s $\Delta\theta$**	**8s $\Delta\theta$**	**4s DE**	**8s DE**	**4s $\Delta\theta$**	**8s $\Delta\theta$**	**4s DE**	**8s DE**
WP	**0.580**	1.362	**1.78**	**2.21**	0.70	1.41	**6.5**	6.8	**0.828**	**1.903**
P1	0.684	1.618	1.85	2.36	0.67	1.28	6.7	7.0	0.832	1.926
P2	0.593	1.295	1.83	2.31	**0.58**	**1.13**	6.7	6.9	**0.826**	**1.899**
P3	0.590	**1.291**	1.82	2.28	**0.59**	1.21	**6.5**	**6.7**	**0.827**	**1.899**
P4	0.595	**1.287**	1.82	2.28	**0.60**	1.26	**6.4**	**6.6**	**0.829**	1.913

8-second prediction:
- WP: models using waypoint representation
- P1-4: using the polynomial representation (degrees 1-4).

Comparable performances with other settings and other models.



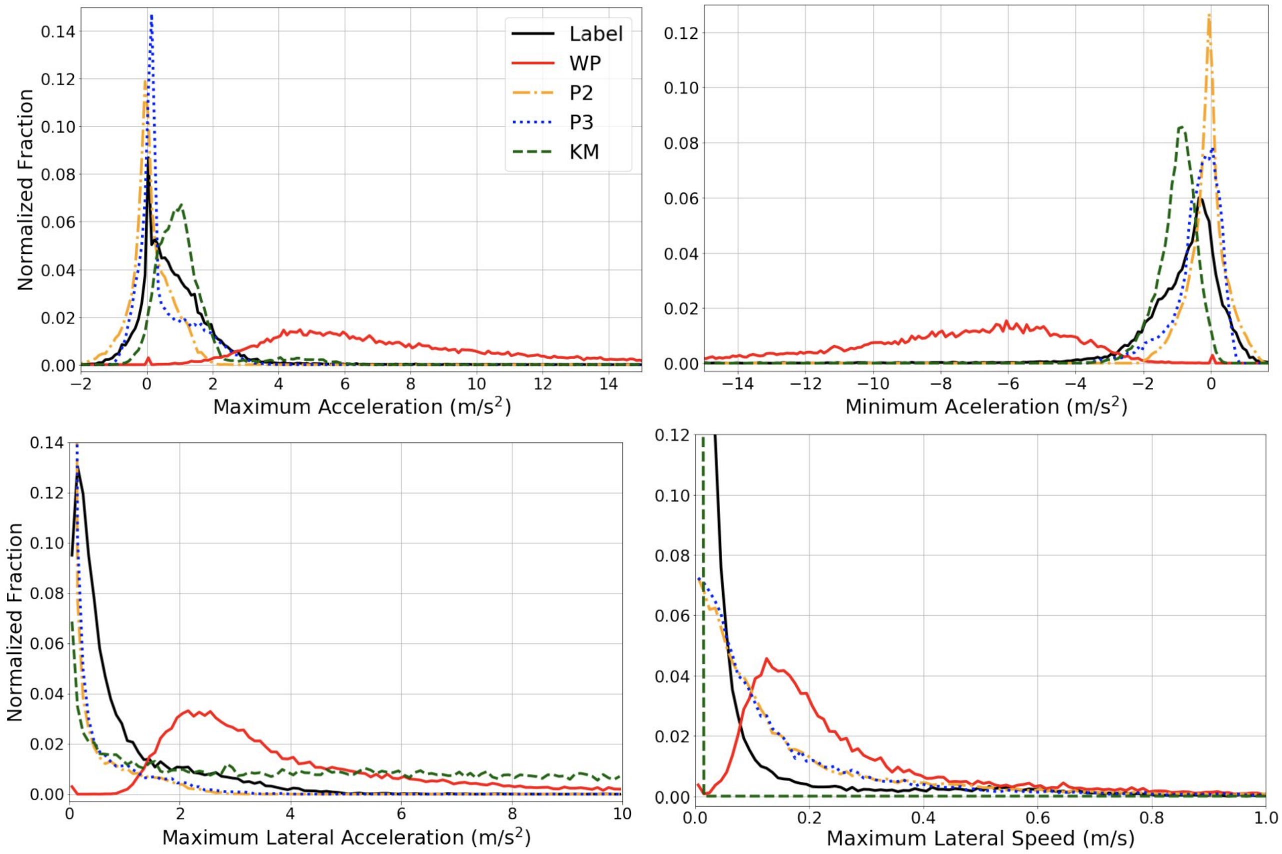
Continuous prediction

	Vehicles				Bicyclists				Pedestrians			
Method	**1s DE**	**2s DE**	**3s DE**	**4s DE**	**1s DE**	**2s DE**	**3s DE**	**4s DE**	**1s DE**	**2s DE**	**3s DE**	**4s DE**
WP	0.99	**1.92**	3.43	4.97	3.9	7.7	11.1	14.2	**0.34**	**0.63**	**0.96**	**1.30**
P2	**0.92**	**1.90**	**3.21**	**4.85**	**2.9**	**5.1**	**6.8**	**8.3**	**0.34**	**0.64**	**0.95**	**1.29**
	1s $\Delta\theta$	**2s $\Delta\theta$**	**3s $\Delta\theta$**	**4s $\Delta\theta$**	**1s $\Delta\theta$**	**2s $\Delta\theta$**	**3s $\Delta\theta$**	**4s $\Delta\theta$**	**1s $\Delta\theta$**	**2s $\Delta\theta$**	**3s $\Delta\theta$**	**4s $\Delta\theta$**
WP	1.97	2.92	4.13	**5.59**	**4.6**	6.6	8.2	9.0	-	-	-	-
P2	**1.87**	**2.81**	**4.06**	**5.56**	**4.6**	**6.4**	**7.9**	**8.7**	-	-	-	-

4-second prediction:

- Regression supervision at 0s, 2s, and 4s
- The predictions of waypoint model (WP) at 1s and 3s are by linear-interpolation;

Physical feasibility



P2-3 achieve physical realism without additional constraints or regularization.

KM: applying vehicle kinematic model on top of the waypoint representation.

Summary (<u>polynomials</u> vs. <u>waypoints</u>)

- Comparable prediction performance
- Calibrated probabilistic predictions
- Better inter-time-point predictions
- Better prediction accuracy for low-count actors
- Better prediction accuracy for large supervision time intervals
- Physical realism without physical models/regularization/constraints