Temporally-Continuous Probabilistic Prediction using Polynomial Trajectory Parameterization

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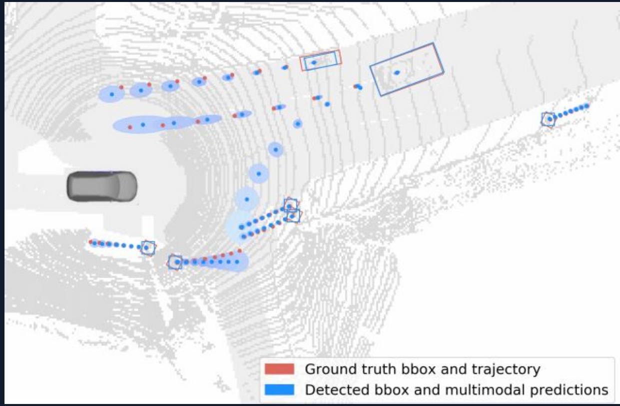
Question

We need to forecast the trajectories of moving actors around the robot.

How to represent the trajectories?

(If framed as a regression task, what actually to regress?)



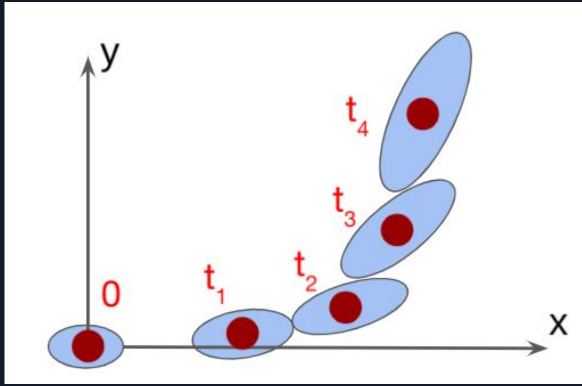
“Default” representation for trajectory forecasting: waypoints

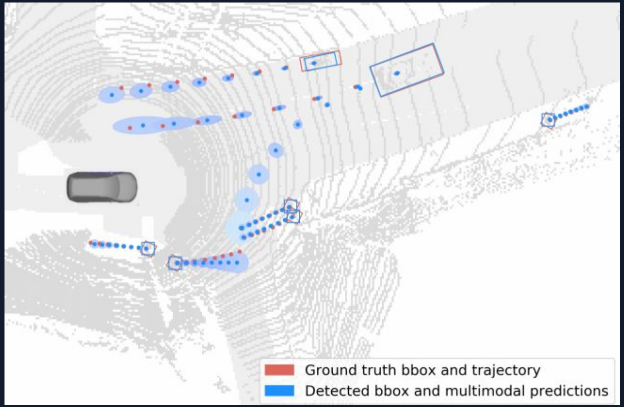
Spatial distributions for the SE3/SE2 transformation at each time-point

$$p_{v,t} = \mathscr{L}\left(v \middle| \mu_{v,t}, b_{v,t}\right), t \in \{0, t_1, t_2, \ldots, T\}$$

v : translation (x, y, z) and the rotation component of SE3/SE2 transformation

μ, b: mean and diversity parameter of distributions (such as Gaussian or Laplacian)





Proposed representation for trajectory forecasting: parameterization

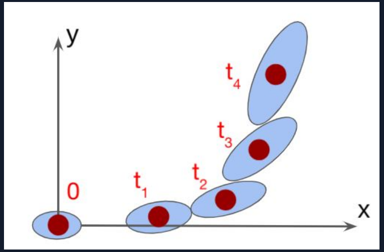
Waypoint representation

$$p_{v,t} = \mathscr{L}\left(v|\mu_{v,t}, b_{v,t}\right), t \in \{0, t_1, t_2, \ldots, T\}$$

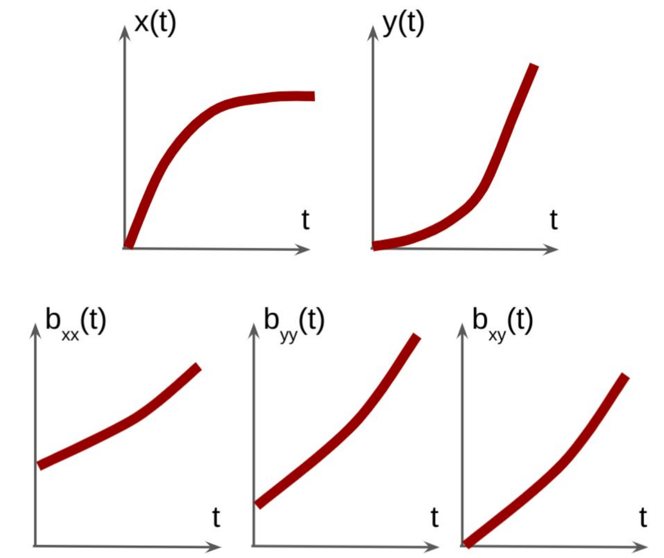
Parametrize the mean and covariance over temporal dimension

Parameterized over time

$$P_v(t) = \mathscr{L}(v|\mu_v(t), b_v(t))$$



2D Laplacian example



Parameterization based on polynomials

$$P_v(t) = \mathscr{L}(v|\mu_v(t), b_v(t))$$

$$\mu_v(t) = \sum_{n=0}^{N_{\mu_v}} a_{\mu_v,n} \left(\frac{t}{T}\right)^n$$

T: maximum prediction time horizon

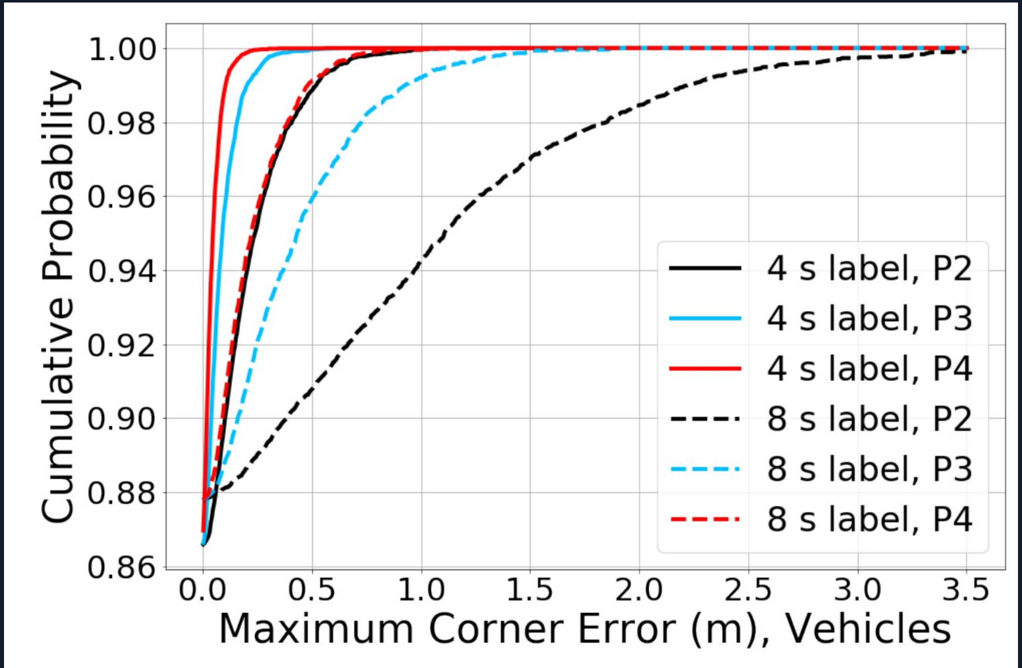
$$b_v(t) = \exp \sum_{n=0}^{N_{b_v}} a_{b_v,n} \left(\frac{t}{T}\right)^n$$

Exponential to ensure positiveness

Polynomial vs. waypoint representation

	Polynomial	*Waypoints*
At pre-fixed time-points	Approximation errors to fit given trajectories	Can describes the distribution means perfectly
Regularization/constraint	Yes, with low-order polynomials	No
Physical realism	Feasible without regularization	Often infeasible
Temporal continuity	Yes; analytical solution	No, unless interpolated
Velocity, acceleration, etc	Analytical solution	Finite differencing

The representation error



How well can low-order polynomials represent label trajectories of ***moving*** actors around autonomous vehicles?

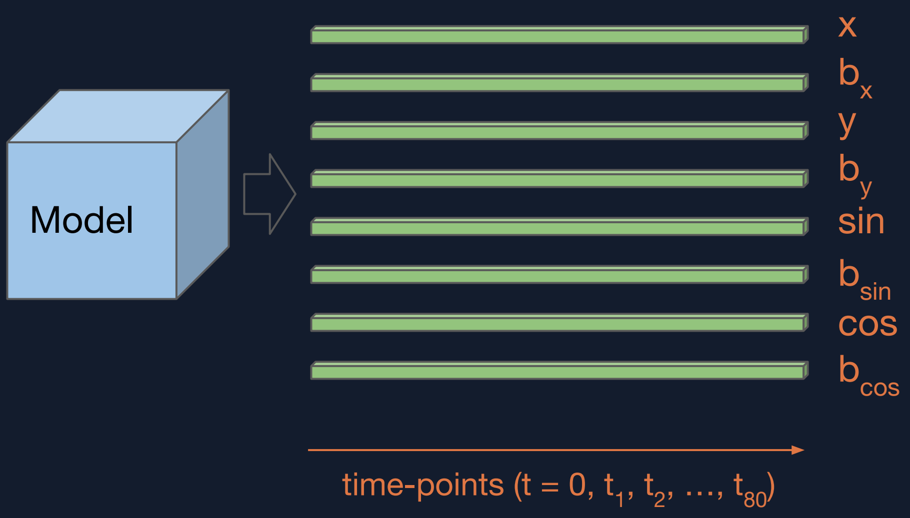
4s label: using polynomials to fit trajectories of **4s** long
8s label: using polynomials to fit trajectories of **8s** long.

Representation errors of polynomials to fit label trajectories. Maximum corner error is the max displacement computed over all four corners and all time-points of the trajectory.

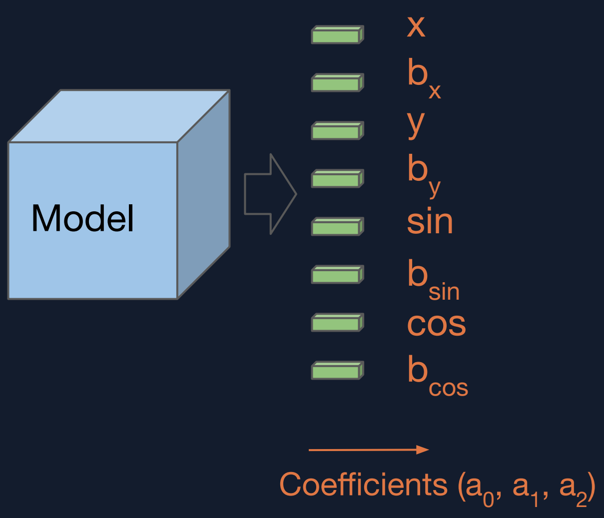
How to use

- ❑ End-to-end training neural networks
- ❑ Identical model design, except for different output representations (i.e., different regression values)

<u>Waypoints:</u>
regress the movements for every time-point



<u>Polynomial:</u>
regress the coefficients



During training waypoints are sampled at the same points $t = 0, t_1, t_2, \ldots, t_{80}$, and the same KL-divergence regression loss is applied

Comparable prediction accuracy (waypoint vs. polynomial)

Ablation studies on SOTA trajectory prediction models that take 10 sweeps past LiDAR data to detect actors and forecast their future trajectories

	Vehicles				Bicyclists				Pedestrians	
Method	**4s DE**	**8s DE**	**4s $\Delta\theta$**	**8s $\Delta\theta$**	**4s DE**	**8s DE**	**4s $\Delta\theta$**	**8s $\Delta\theta$**	**4s DE**	**8s DE**
WP	0.580	1.362	1.78	2.21	0.70	1.41	6.5	6.8	0.828	1.903
P3	0.590	1.291	1.82	2.28	0.59	1.21	6.5	6.7	0.827	1.899

Comparable

Better for less common type

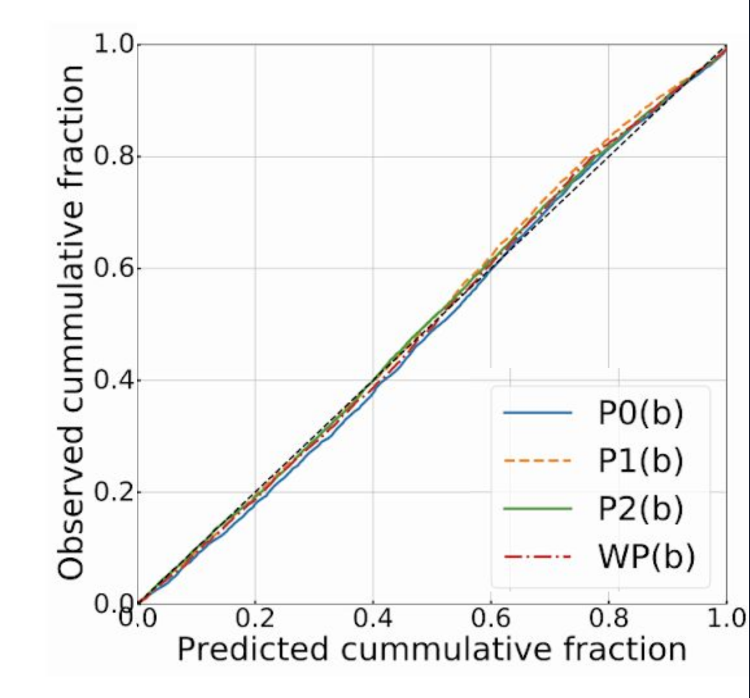
Comparable

8-second prediction:

- WP: models using waypoint representation
- P3: using the polynomial representation (degree 3).

Comparable performance relative to other settings (4s) and other models too (i.e., those with different architectures and regression losses; detailed results shown in the paper).

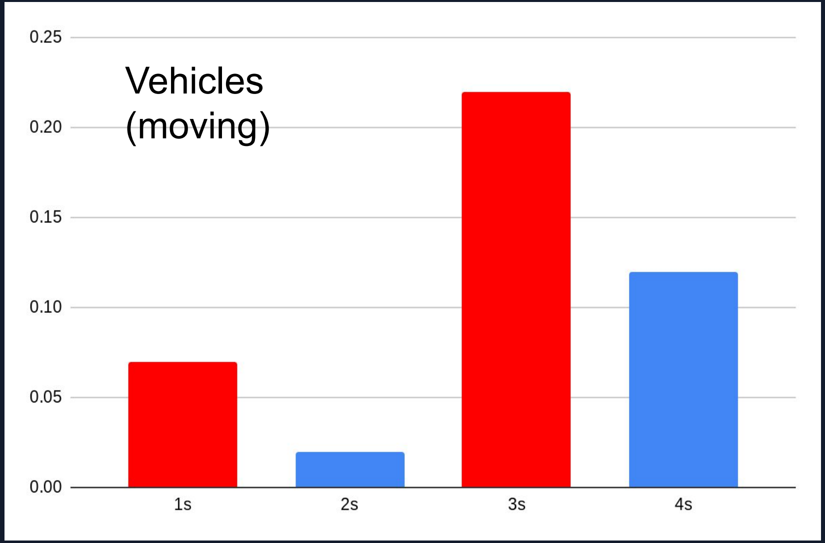
Comparable calibration of probabilistic prediction

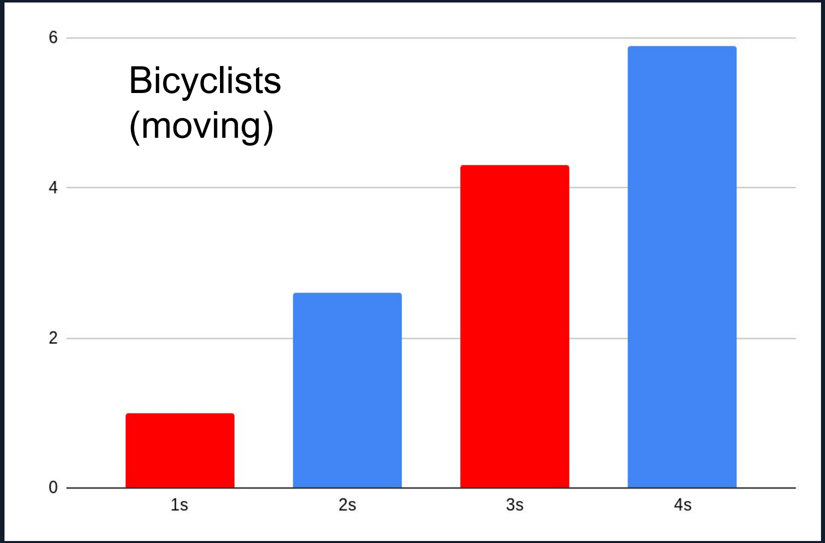


Probably calibration reliability diagram of models using waypoints (WP(b)) and polynomials of degrees 0-2 (P0-2(b)) for the distribution diversity parameters.

Continuous prediction vs. interpolation

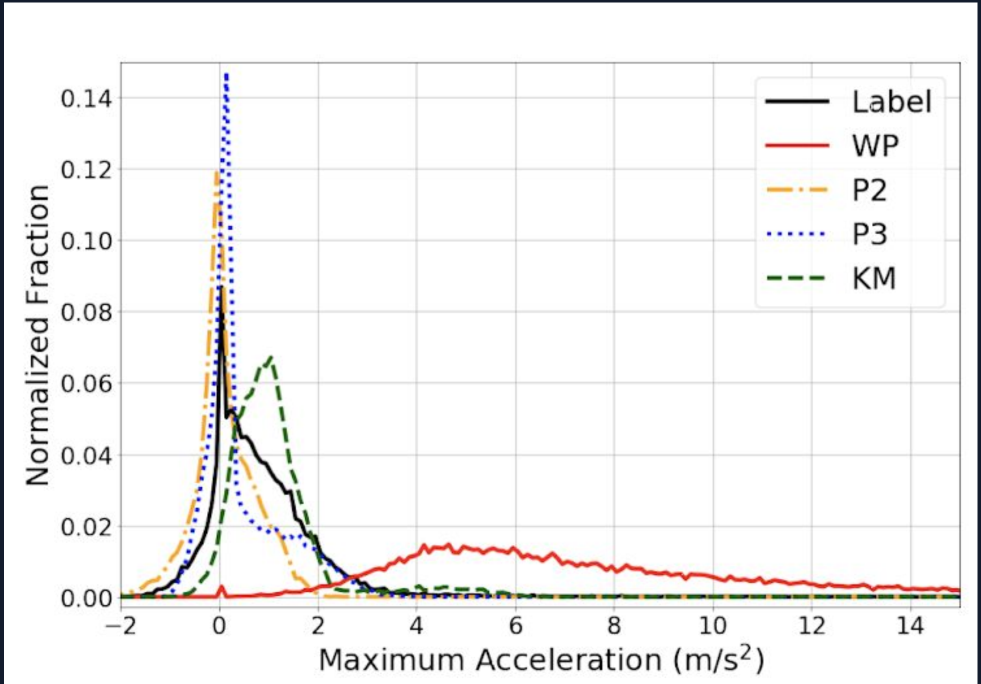
Displacement error improvement using polynomials over waypoints (in meters)





- Regression supervision only at 0s, 2s, and 4s. The validation performance are in blue bars.
- No regression supervision at 1s and 3s. The validation performance are in red bars.
- The predictions of waypoint model at 1s and 3s are computed by linear interpolation

Physical feasibility of inferred trajectories



WP (red): waypoints representation is physically unrealistic, compared to label trajectories (**black**).

Polynomials (P2-3, **yellow** and **blue**, resp.) achieve physical realism without additional constraints or regularization.

KM (green): vehicle kinematic model at waypoints + regularization

Similar holds for deceleration, lateral speed, and lateral acceleration (results not shown).

Contributions

We proposed a polynomial representation for trajectory forecasting

- Temporally continuous and compact
- Beneficial regularization for low-count actors and/or sparser temporal supervision
- Increased physical realism without physical models or additional regularization
- Comparable prediction accuracy
- Calibrated probabilistic prediction